Appendix for: Allocating Costly Influence in Legislatures

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Supplementary models

Within-chamber type results are given in Table 1 below. Unit random intercepts are still allowed as in the central main text results. These results differ very little from the main text model.

Table 1: Model including chamber type fixed effects.

Constant	0.234	(0.024)
Minority seat share	0.179	(0.056)
Committee size	0.014	(0.004)
Majority cohesiveness	−0.056	(0.034)
Concentration of procedural power	0.028	(0.004)
Pattern of competition	−0.120	(0.116)
Proportionality rule	−0.078	(0.028)
Minority conference rule	−0.100	(0.021)
Minority seats × competition	0.386	(0.278)
Minority seats × proportionality	0.213	(0.080)
Minority seats × minority conference	0.224	(0.054)
Upper chamber	0.008	(0.004)
Observations	2,144	
Log Likelihood	2,675.524	

Table 2 gives results from a random effects model estimated via MCMC. They are effectively identical the main text MLE results.

Table 2: Main model as estimated via Markov chain.

	Mean	SD
Constant	0.255	0.022
Minority seat share	0.171	0.057
Committee size	0.010	0.003
Majority cohesiveness	-0.044	0.033
Concentration of procedural power	0.027	0.004
Pattern of competition	-0.197	0.106
Proportionality rule	-0.072	0.027
Minority conference rule	-0.098	0.021
Minority seats × competition	0.567	0.255
Minority seats × proportionality	0.190	0.079
Minority seats × minority conference	0.215	0.054
Observations	2,144	
Log Likelihood	2,675.524	

A Model of Repeated Dictator Game as Applied to Legislative Committee Seat Assignment

This note shows how reciprocity can emerge in equilibrium in repeated interactions of legislative committee seat assignment, particularly when the majority party's hold over the legislature is tenuous, as predicted in the paper. By reciprocity we mean the majority party assigns a more equitable number of seats to the minority party in the current period than is required to maximize the majority party's payoff in the current period, and the current minority party will return the favor in the future when it becomes the majority party.

The setup is an infinitely repeated game with a majority party and a minority party. Let time periods be indexed by $t = 0, 1, 2, \ldots$ The two players are both risk neutral and discount the future by a common discounter factor $\sigma < 1$. In each period, the majority party is the only active player and dictates committee seat assignments, and the minority party passively accepts the assignments. With probability p, the current majority party will retain its majority status in the next period; with probability $1 - p$ the current minority party will become the majority and the dictator in assigning committee seats.

In each period, the majority party has two choices: A) assigning the number of committee seats to the minority in a way that maximizes the utility of the majority party in the current period (as described in the paper), and B) assigning more seats to the minority than A would require, in the hope that the minority party would return the favor when it becomes the majority party in the future. Choice A will give the majority party a payoff of m in the current period, while choice B will give the majority a payoff of r in the current period, $m > r$ (m stands for maximum and r stands for reciprocity). The minority party's payoff in the current period when the majority chooses A is normalized to be zero, and its payoff in the current period when the majority party gives out more seats to the minority is s (s stands for small). Since it is better to be a majority than a minority party, $m > r > s > 0$.

We will show that there is an equilibrium in which each party will choose B and receive payoff r in each period it is the majority, rather than choosing A. Consider the following grim trigger style strategy for the majority party (a party can only passively accept assignments when it is the minority): Choose B at time period 0. In all future periods, continue to choose

B as long as each party has always chosen B whenever it is the majority party, but choose A if any majority party has ever chosen A in the past. If both parties in the legislature adopt this strategy, then any party that is the majority will always choose B and reciprocity is sustained.

The part of the strategy involving A is easy to understand: if one of the parties will choose A, (thus minimizing the other party's current period payoff) and stick to the strategy from that point on, it makes no sense for the other party to ever choose B, since this choice will not be reciprocated. So if one of the parties will choose A, the other party should always choose A too. We next show that if one of the parties (call it D) has chosen B and stuck to the above strategy, namely continuing to choose B whenever it is the majority party as long as each party has always chosen B in the past (but deviating to A forever if any party ever chooses A), then the other party (call it R) will also reciprocate by always choosing B when it is the majority party.

To see this, assume R is the majority party at time t, and it is contemplating whether to choose A or B. R's continuation value at time t, or its expected payoff in the game from that point on, is the sum of its payoff from the current period and the discounted value of its continuation value when the game advances to the next period. The continuation value of the next period depends on whether the party is the majority or minority party, and whether it has chosen A or B in the current period. If R chooses A at time t to realize the current period payoff m, its continuation value, denoted as V_m, must satisfy the following Bellman equation:

$$V_m = m + \sigma(pV_m + (1-p)V_0) \tag{1}$$

where V_0 indicates the minority party's continuation value when the majority party chooses A, thus leaving payoff 0 to the minority party in the current period. The second term of equation (1)'s right hand side reflects the fact that at time $t+1$, with probability p, R is still the majority party, and it will continue to have the continuation value V_m; with probability $1-p$, R becomes the minority party and its continuation value will be V_0 since

D will retaliate by choosing A from that point on.

Similarly, the continuation value V_0 must satisfy the following Bellman equation:

$$V_0 = 0 + \sigma((1-p)V_m + pV_0) \tag{2}$$

This equation says that, when the majority party chooses A from the current period on, the minority party will get 0 in the current period, and its continuation value will be V_m in the next period with probability $1-p$ (since the majority party will retain its majority status in the next period with probability p), and V_0 with probability p.

Solving equations (1) and (2) together yields the following value for V_m:

$$V_m = \frac{m}{1 - \sigma p - \frac{\sigma^2(1-p)^2}{1-\sigma p}} \tag{3}$$

If R chooses B instead, it will receive payoff r in the current period and its continuation value will be:

$$V_r = r + \sigma(pV_r + (1-p)V_s \tag{4}$$

where V_s is the continuation value of a minority party when the majority party sticks to choosing B. The second term of equation (4)'s right hand side reflects the fact that in the next period, with probability p, R will continue to be the majority party and receive continuation value V_r, and with probability $1-p$, it will become the minority party and have the continuation value V_s, since D will reciprocate by choosing B.

The continuation value V_s can similarly be written as follows:

$$V_s = s + \sigma((1-p)V_r + pV_s) \tag{5}$$

Solving equations (4) and (5) together yields the following value for V_r:

$$V_r = \frac{r + \frac{\sigma(1-p)s}{1-\sigma p}}{1 - \sigma p - \frac{\sigma^2(1-p)^2}{1-\sigma p}} \tag{6}$$

Comparing (3) and (6), we know $V_r > V_m$ if

$$r + \frac{\sigma(1-p)s}{1-\sigma p} > m \tag{7}$$

Taking the derivative of the left hand side of inequality (7) respectively with regards to σ and p shows that the left hand side is increasing in σ but decreasing in p. In other words, $V_r > V_m$ will more likely to hold when the players are patient and when the probability that the majority party will retain its majority status in the next period is low. Inequality (7) is also more likely to be satisfied when the reciprocity payoffs r and s are not too small as compared to the non-reciprocity payoff m, otherwise m will be too attractive for the majority party to moderate its seat assignments.

Summing up the above analysis leads to the following result:

Proposition 1: The grim trigger style strategy can sustain reciprocity in equilibrium, in which the party in the majority will assign more seats to the minority party than maximizing the former's current period payoff would require, if $r + \frac{\sigma(1-p)s}{1-\sigma p} > m$. The condition is more likely to hold when the reciprocity payoffs r and s are relatively large, the discount factor σ is high, and the majority party's probability of retaining its majority status p is low.